The 8th Workshop "Complex Geometry and Lie Groups"

March 10th - 14th, 2025, Osaka, Japan

Abstract

Dan Popovici: *Higher-degree holomorphic contact structures.*

This is joint work with H. Kasuya and L. Ugarte. We introduce the classes of holomorphic p-contact manifolds and holomorphic s-symplectic manifolds that generalise the classical holomorphic contact and holomorphic symplectic structures. After observing their basic properties and exhibiting a wide range of examples, we give three types of general conceptual results involving the former class of manifolds: structure theorems; hyperbolicity results; unobstructedness theorems, generalising to our context the classical Bogomolov-Tian-Todorov theorem, for two types of small deformations of complex structures that generalise the small essential deformations previously introduced for the Iwasawa manifold and for Calabi-Yau page-1-$\partial\bar{\partial}$-manifolds.

Adriano Tomassini: *Non-Kähler manifolds with special structures.*

We will present some recent results on the existence of special structures on compact non-Kähler manifolds obtained as quotients of $\mathbb{C}^n \ltimes \mathbb{C}^m$. More in particular, we will focus on *p-Kähler structures*, on the *$\partial\bar{\partial}$-Lemma*, on symplectic structures satisfying the *Hard Lefschetz Condition* and on some formality properties. The results discussed have been obtained in some papers joint with A. Cattaneo, E. Lo Giudice, F. Lusetti, T. Sferruzza and L. Sillari.

Giovanni Bazzoni: *Complex symplectic structures on Lie groups.*

Complex symplectic structures are the analogue of real symplectic structures in the context of complex geometry. They are related to different metric structures, such as hyperkähler and hypersymplectic, and play a role in Physics, for instance in holomorphic Hamiltonian systems. In this talk I will describe some techniques for constructing complex symplectic structures on Lie groups, with particular emphasis on the nilpotent case. Furthermore, I will provide some results on their stability under deformation.

Hideyuki Ishi: *The Tian-Yau-Zeldich expansions for homogeneous Kähler metrics on a boundod homogeneous domain.*

Let g be a polarized Kähler metric with respect to a holomorphic line bundle L over a complex manifold M. Then the pull-back of the Fubini-Study metric by the Kodaira map associated to the m-th tensor power of L gives an approximation of the metric g when m goes to infinity. Kempf's distortion function T_m of g indicates the difference of the approximation. The assymptotic expansion of T_m with respect to the parameter m is called the Tian-Yau-Zeldich expansion. In this talk, we compute the function T_m explicitly for any homogeneous Kähler metric on a bounded homogeneous domain of dimension n. As a matter of fact, T_m is a polynomial of degree n with rational roots.

Mario Garcia Fernandez: *Commuting pairs of $N = 2$ algebras from canonical generalized Kähler* structures and the elliptic genus.

We construct commuting pairs of the N = 2 superconformal vertex algebra in the chiral de Rham complex of a generalized Kähler manifold. Our construction requires that the generalized Kähler structure is "canonical", in a precise sense, covering an important class of compact non-Kähler examples, such as compact even dimensional Lie groups. Using this, we propose a natural notion of elliptic genus for generalized Kähler manifolds, following work by Heluani and Zabzine on this problem. Extensions of the physical Bershadsky-Cecotti-Ooguri-Vafa (BCOV) conjecture, relate our elliptic genus with higher genus Gromov-Witten invariants on the mirror manifold. Joint work with Andoni de Arriba de la Hera, Luis Alvarez-Consul, and Jethro van Ekeren.

Vestislav Apostolov: *From Kähler-Ricci solitons to Calabi-Yau cones.*

In this talk I will discuss the proof of the following recent result obtained in collaboration with Lahdili and Legendre: if X is a smooth Fano manifold which caries a Kähler-Ricci soliton, then the canonical cone of the product of X with a complex projective space of sufficiently large dimension is a Calabi-Yau cone. This can be seen as an asymptotic version of a conjecture by Mabuchi and Nikagawa. The result is obtained using the openness of the set of weight functions v over the momentum polytope of a given smooth Fano manifold, for which a v-soliton exists. Time permitting, I will discuss other ramifications of this approach.

Eleonora Di Nezza: *Quasi-convexity of the Mabuchi Functional in singular settings.*

The Mabuchi functional M was introduced by Mabuchi in the 80's in relation to the existence of canonical metrics on a compact Kähler manifold. The critical points of M are indeed constant scalar curvature Kähler (cscK) metrics. Recently, Chen and Cheng proved that the existence of a (smooth) cscK metric is equivalent to the properness of such functional. In order to look for singular metrics, it is then natural to study the properties of the Mabuchi functional in singular settings. In this talk we prove that this functional is (almost) convex in the very general "big case".

Scott Wilson: *On the interplay between topology, geometry, and complex analysis on compact manifolds.*

The following two questions are prominent and seemingly difficult: on compact manifolds, are there topological obstructions to the existence of complex structures, and secondly, what are the complex-analytic obstructions to various special metrics? In this talk, I will discuss a new way to approach these questions, for certain types of complex structures. A key idea is to organize complex manifolds according to the structure of their differential forms, by generalizing some classical work in the 1970's on Kähler manifolds and the $\partial\bar{\partial}$-Lemma. Proceeding in this way, we find that there are indeed obstructions in each of the cases mentioned above, which are easy to understand. In particular, there are computable invariants of the cohomology ring and rational homotopy type that prohibit the existence of various types of complex structures, even on quotients of nilpotent Lie groups. Applications to the study of Vaisman metrics, and to stability under deformations of complex structures, will be given as well. This is joint work with Jonas Stelzig.

Luigi Vezzoni *New advances in HyperHermitian geometry.*

The talk focuses on special metrics which are compatible with hypercomplex structures. In particular, we will consider hyperKähler with torsion metrics (HKT) introduced by Howe and Papadopoulos in 1996. These metrics represent for many aspects the hypercomplex counterpart of Kähler manifolds, and in this context, balanced HKT metrics have the role that Calabi-Yau metrics have in Kähler geometry. In 2010 Alesker and Verbitsky introduced a Calabi-Yau type problem in HKT geometry, stating a conjecture that is still open. In analogy to the classical Calabi-Yau theorem, the problem introduced by Alesker and Verbitsky brings to the study of a Monge-Ampère type equation on hyperKähler manifolds and, more generally, to the study of fully nonlinear elliptic and parabolic equations. In the talk I will present some progresses in the study of such equations I have obtained in collaboration with Lucio Bedulli and Giovanni Gentili.

Hiroshi Tamaru: *Nilpotent Lie algebras obtained by quivers and Ricci solitons.*

Nilpotent Lie groups with left-invariant metrics provide non-trivial examples of Ricci solitons. In this talk, we use quivers to construct nilpotent Lie algebras. A quiver is a directed graph that allows loops and multiple arrows between vertices. Utilizing the concept of paths within quivers, we introduce a method for constructing nilpotent Lie algebras from finite quivers without cycles. We prove that for all these Lie algebras, the corresponding simply-connected nilpotent Lie groups admit left-invariant Ricci solitons. The method we present constructs a broad family of Ricci soliton nilmanifolds with arbitrarily high degrees of nilpotency. Additionally, we mention some recent progress related to this topic. This work is based on collaboration with Fumika Mizuguchi.

Ramiro Lafuente: *Collapsing Ricci flows and Higgs bundles.*

In this talk I will discuss recent joint work with Adam Thompson in which Lie groups and holomorphic objects (Higgs bundles) appear naturally. We describe the asymptotic limits of immortal Ricci flows in dimensions 4 and above that collapse (with bounded curvature and diameter) to a surface. This is done by studying expanding Ricci solitons on vector bundles over closed surfaces, where the fibres admit local isometric actions by a nilpotent Lie group. We reduce the soliton equation to the equivariant harmonic-Einstein equations, and we solve the latter by establishing a close link with the theory of Higgs bundles.

Takayuki Okuda: *Coarse coding theory and discontinuous groups on homogeneous spaces.*

Let M and $\mathcal{I}$ be both sets, and fix a surjective map

$$R : M \times M \to \mathcal{I}.$$

Then, for each subset $\mathcal{A}$ of $\mathcal{I}$, $\mathcal{A}$-free-codes on M are defined as subsets C of M with $R(C \times C) \cap \mathcal{A} = \emptyset$. This definition of codes encompasses error-correcting codes, spherical codes, and other codes defined on association schemes or homogeneous spaces.

In this talk, we fix a "generalized-bornological coarse structure" on the set $\mathcal{I}$, and give a definition of "coarsely $\mathcal{A}$-free-codes" on M. This provides a generalization of the concept of $\mathcal{A}$-free-codes as mentioned above. As the main result, we will discuss the relationships between coarse coding theory on Riemannian homogeneous spaces $M = G/K$ and discontinuous group theory on non-Riemannian homogeneous spaces $X = G/H$.

Jeffrey Streets: *Generalized Ricci Flow and the Hull-Strominger System.*

The Hull-Strominger system was introduced in 1986 in physics as a geometric model for string backgrounds, and was proposed by Yau as a tool for geometrizing Reid's fantasy. In this talk I will describe a geometric flow approach to solving for this system arising from the theory of generalized Ricci flow/pluriclosed flow. A key result is to give a natural interpretation of this flow in terms of the geometry of a holomorphic string algebroid. As a consequence of this we obtain a natural extension of Yau's C^3 estimate for the complex Monge-Ampere equation, and global existence/convergence results for the flow. Joint work with M. Garcia-Fernandez and R. Gonzalez Molina.

Yuta Kusakabe: *On pseudoconvexity of Gromov elliptic manifolds.*

A complex manifold X is said to be Gromov elliptic if it admits a dominating holomorphic spray, which can be regarded as a holomorphic family of dominating entire holomorphic maps $\mathbb{C}^n \to X$. In contrast to Kobayashi hyperbolic manifolds, Gromov elliptic manifolds admit an abundance of entire holomorphic maps. Typical examples include complex Lie groups, and it is known that every Gromov elliptic manifold is an Oka manifold. In this talk, we generalize the Adachi–Suzuki–Yosida theorem on the Hartogs extension property of complex Lie groups. Our main result states that every Gromov elliptic manifold X enjoys the Hartogs extension property with respect to the q-Hartogs figure for any q greater than the nullity $n - \dim X$ of a dominating spray over X. Furthermore, using the same technique, we construct several compact counterexamples to Gromov's question on Gromov ellipticity of Oka manifolds.

Eveline Legendre: *The CR-Yamabe invariant and constant scalar curvature metrics on polarized manifolds.*

This is a joint work with Abdellah Lahdili and Carlo Scarpa, where we explore further the outcomes of a correspondence, we have previously established, between the constant scalar curvature Kähler (cscK) problem on a polarized manifold and a family of CR-Yamabe problems on the associated circle bundle. Among other things, we show that in some cases, the supremum of the associated CR-Yamabe invariants detects approximate cscK metrics and K-semistability.

Yoshihiko Matsumoto: *A construction of Poincaré-Einstein metrics of cohomogeneity one on the ball.*

After explaining some generalities on Poincaré-Einstein metrics, I would like to present an elementary, explicit construction of a one-parameter family of PE metrics on the even-dimensional ball whose conformal infinities are the Berger spheres, which is based on an ansatz of Page and Pope. I will then explain how this family converges to the complex hyperbolic metric, thereby providing a continuous family of Einstein metrics interpolating the real and the complex hyperbolic metrics. This convergence becomes apparent only when an appropriate gauge is chosen, i.e., when each metric is pulled back by a suitable diffeomorphism from the ball. If possible, I would also like to discuss how such a gauge choice can be found geometrically.

Vicente Cortés: *Darboux theorem for generalized complex structures on transitive Courant algebroids.*

I will present a theorem describing the local form of generalized complex structures on transitive Courant algebroids under appropriate local regularity conditions. The result extends Gualtieri's Darboux theorem for generalized complex structures on exact Courant algebroids and at the same time Wang's description of skew-symmetric left-invariant complex structures on compact semi-simple Lie groups. The talk is based on joint work with Liana David (arXiv:2501.03669).