Workshop "Complex Geometry and Lie Groups"
March 10 -14, 2025, Osaka, JAPAN

Short Communication Session

1 **********
Your Name：Satoshi Nakamura
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Title: Continuity method for the Mabuchi soliton on the extremal Fano manifolds.

Abstract: We run the continuity method for Mabuchi's generalization of Kähler-Einstein metrics, assuming the existence of an extremal Kähler metric. It gives an analytic proof (without minimal model program) of the recent existence result obtained by Apostolov, Lahdili and Nitta. Our key observation is the boundedness of an energy functional along the continuity method. This talk is based on arXiv:2409.00886, the joint work with Tomoyuki Hisamoto (Tokyo Metropolitan University).

2 **********
Your Name：Taito Shimoji
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Title: Counterexamples to Serre’s problem.

Abstract: "Can every finitely generated group be realized as fundamental groups of smooth complex algebraic varieties?" is known as "Serre's problem". In this talk, we illustrate a method of obtaining a counterexample to Serre's problem. The method is to use the Morgan's mixed Hodge structure, which reflects an information on fundamental groups of algebraic varieties to graded nilpotent Lie algebras. Moreover, using a relation between nilpotent Lie groups and rational Lie algebras, we introduce our result that a class of lattices of some simply connected Lie groups is counter examples to Serre's problem.

3 **********
Your Name：Stanley Luk
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Title: The Cartan Development of the Signature of a Path.

Abstract: Consider a path in the n-dimensional Euclidean space V. The collection of all iterated integrals of the path components is called the Signature of that path. It is used to identify the path with a formal power series in the tensor algebra T(V). This identification was first studied in the ‘50s by K. T. Chen, who viewed such power series as a formal representative of the paths themselves. This idea is greatly extended in the ‘90s by the Rough Path Theory invented by T. Lyons where the signature is adapted to study highly irregular paths such as the sample paths of a Brownian motion. In this talk I will introduce a concept important to rough path theory, the Cartan Development of a Signature of Path, but we mostly work in the smooth paths in V case for simplicity. For a Lie group G with Lie algebra g, the Cartan development maps the signature of a path into the algebra g and, through some more steps, associates to original path an element of the Lie group G. (The dimension of G does not need to be n.) By choosing a specific Lie group G, along with a representation of G, for us to develop our path into, we can pick up geometric information about our path in V. This idea has already been successfully used to determine geometric and analytic properties about some classes of paths, both smooth and irregular. The philosophy is that by suitably varying the representations of G, or even the Lie group G itself, in our Cartan development allows us to determine all information about the original path. Although we are still some way off from that theoretical ideal, we present this with the hope that experts listening might have fresh ideas about which Lie groups we could use to Cartan develop our paths into and what new properties they might pick up about the underlying path.

4 **********
Your Name：Shuho Kanda
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Title: On meta-abelian LCK solvmanifolds and Oeljeklaus-Toma manifolds.

Abstract: When studying a certain class of objects, examples with structures that are simple enough for computations while being complicated enough to exhibit non-trivial properties are often highly valued. In the context of LCK manifolds, Oeljeklaus-Toma (OT) manifolds serve as such examples. These manifolds are constructed using number-theoretic methods, possess a meta-abelian solvmanifold structure, and yet exhibit a complicated structure as LCK manifolds. This raises the question: conversely, are meta-abelian solvmanifolds that admit LCK structures limited to OT manifolds? By

exploring number-theoretic properties arising from lattices in Lie groups, we obtained partial results for this question.

5 **********
Your Name：Roberto de Santana Araujo
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Title: Ricci flows on awesome standard homogeneous vector bundles.

Abstract: The Dynamical Alekseevskii Conjecture states that if a simply connected homogeneous manifold is not contractible, then all homogeneous Ricci flow solutions on it have finite extinction time. Böhm showed that the conjecture is true for compact homogeneous spaces. In this talk, we will discuss the conjecture in the context of noncompact homogeneous manifolds. We will be especially concerned with the case where the Lie algebra of the transitive group of isometries is a semidirect product of a compact Lie algebra and an abelian ideal. We will then show the conjecture to be true on a family of initial homogeneous metrics that satisfy some compatibility condition between the Lie algebraic structure and its geometry.

6 **********
Your Name：Adjaratou Arame Diaw
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Title: On equivariant \mathbb {C}^2- embeddings.

Abstract: In this talk, we are going to give the local classification of an \mathbb {C}^2-action on a complexe surface. Moreover, we will see how to glue these local actions in order to obtain a global action having a dense open orbit isomorphic to \mathbb {C}^2.

7 **********
Your Name：Alejandro Tolcachier
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Title: (Non)-existence of hypercomplex structures on special linear groups.

Abstract: Hypercomplex manifolds are quaternionic analogues of complex manifolds: they are differentiable manifolds equipped with a hypercomplex structure (HCS), that is, a triple of complex structures $\{J_1, J_2, J_3\}$ satisfying the quaternion laws: $J_1 J_2=-J_2 J_1=J_3$. Such a manifold has real dimension $4n$, $n\in\mathbb{N}$. Any hypercomplex manifold admits a unique torsion-free connection ∇^{Ob} such that $\nabla^{Ob} J_\alpha=0$ for $\alpha=1,2,3$. This connection is called the Obata connection and its holonomy group is contained in $\operatorname{GL}(n,\mathbb{H})$. An important source of examples of hypercomplex manifolds is provided by compact Lie groups. Indeed, in 1992 Joyce proved that any $4n$-dimensional compact Lie group admits a left-invariant HCS. In the non-compact case, there are several results about HCSs on nilpotent and solvable Lie groups. However, there are no general results about HCSs on non-compact semisimple Lie groups. In this talk we will show that a natural candidate, the 8-dimensional Lie group $\operatorname{SL}(3,\mathbb{R})$ does not admit any left-invariant HCS. On the other hand, we will see that $\operatorname{SL}(2n+1,\mathbb{C})$ does admit a left-invariant HCS for any $n\in\mathbb{N}$ and we will determine explicitly its Obata holonomy group, which is properly contained in $\operatorname{GL}(m,\mathbb{H})$ and not contained in $\operatorname{SL}(m,\mathbb{H})$, where $4m=\dim_{\mathbb{R}} \operatorname{SL}(2n+1,\mathbb{C})$. This talk is based on a joint work with Adrián Andrada and Agustín Garrone (Universidad Nacional de Córdoba, Argentina).

8 **********
Your Name：Asia Mainenti
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Title: p-Kähler and Hodge-Riemann balanced structures on solvmanifolds.

Abstract: A p-Kähler manifold is a complex manifold endowed with a p-Kähler structure, namely a closed transverse (p, p)-form. For the extremal values of p, we get the well known Kähler and balanced manifolds, however, for each other possible p, such structures have no metric meaning. The aim of the talk, after discussing similarities and differences between p-Kähler structures and Kähler and balanced structures, is to present some results on the existence of p-Kähler structures on solvmanifolds, with a focus on low dimension. The second part will address Hodge-Riemann balanced metrics, introduced by X. Chen and R. Wentworth. We will examine the relation between such structures and p-Kähler ones, and find non-existence results of Hodge-Riemann balanced metrics on several classes of non-Kähler solvmanifolds. To conclude, we construct the first non-Kähler example of a Hodge-Riemann balanced structure, on a non-compact complex manifold obtained as the product of the Iwasawa manifold by C. This is joint work with A. Fino.

9 **********
Your Name：Tommaso Sferruzza
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Title: "Cohomology and geometric formality of Kähler solvmanifolds".

Abstract: "In this short talk, based on a joint work with Adriano Tomassini, I will prove a characterization of the cohomology of Kähler solvmanifolds. As a consequence, I will describe explicitly their space of harmonic forms and prove that these manifolds are always geometrically formal".

10 **********
Your Name：Satoshi Jinnouchi
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Title: Slope Stable Sheaf and Hermitian Einstein Metric on Normal Varieties with Big Cohomology Classes.

Abstract: Main theme of this talk is the Kobayashi-Hitchin correspondence on normal varieties with big classes. Kobayashi-Hitchin correspondence is the one to one correspondence between slope stable sheaves and Hermitian-Einstein sheaves. This is classically studied on K¨ahler manifolds. In this talk, I generalize the notion of slope stability and Hermitian-Einstein metric for normal varieties with big classes. I will also explain the Kobayashi-Hitchin correspondence on normal projective varieties of general type with respect to the canonical divisor. The bimeromorphic invariance of these notions is one of the key facts for proof.

11 **********
Your Name：Takahiro Aoi
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Title : Microscopic stability thresholds and constant scalar curvature Kähler metrics.

Abstract : For Fano manifolds, Berman defined uniform Gibbs stability by introducing the microscopic stability thresholds. Fujita-Odaka proved that if a Fano manifold is uniform Gibbs stable then it admits a Kähler-Einstein metric by a purely algebro-geometric way. On the other hand, Berman gave an analytic and direct proof of this result. In this talk, I will explain about a generalization of Berman's result to constant scalar curvature Kähler metrics on polarized manifolds. The existence result for a constant scalar curvature Kähler metric with cone singularities along a divisor is also given.

12 **********
Your Name：Shunsuke Saito
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Title: K-instability of hyperplane sections of Segre varieties.

Abstract: In the early 1980s, Sakane and Hano proved that the automorphism group of a smooth hyperplane section of the Segre variety $\Sigma_{m, n}$ is nonreductive when $m\neq n$, and consequently revealed that it does not admit a constant scalar curvature K\"ahler metric in any K\"ahler class. According to the famous Yau-Tian-Donaldson conjecture, which is now a central topic in K\"ahler geometry, the existence of constant scalar curvature K\"ahler metrics should be equivalent to an algebro-geometric condition "K-polystability". Therefore, such hyperplane sections are expected not to be K-polystable. In this talk, I will explain that the following statement holds, which is an algebro-geometric counterpart of Sakane and Hano's results: a normal hyperplane section of the Segre variety $\Sigma_{m, n}$ are K-unstable for any polarization either when $m\neq n$ or when it is singular.

13 **********
Your Name：Elia Fusi
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Title: Blowing up balanced Chern-Ricci flat metrics.

Abstract: In 2006, Arezzo and Pacard proved the existence of cscK metrics on the blow-up at finitely many points of a cscK manifold admitting no non-trivial holomorphic vector fields. In this talk, I will describe the gluing strategy we used to construct balanced constant Chern scalar curvature metrics on the blow-up of a compact balanced Chern-Ricci flat manifold. Finally, I will explain how the same gluing strategy can be adapted to construct Chern-Ricci flat metrics on crepant resolutions of Chern-Ricci flat orbifolds. This is a joint work with Federico Giusti.

14 **********
Your Name：Natsuo Miyatake
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Title: Towards a Statistical-Mechanical Formalism for Harmonic Metrics on Cyclic Higgs Bundles.

Abstract: Let X be a Riemann surface, and let $K_X \to X$ denote its canonical bundle. For each integer $r \geq 2$, each $q \in H^0(K_X^r)$, and each choice of a square root $K_X^{1/2}$ of the canonical bundle, we can canonically construct a Higgs bundle, referred to as a cyclic Higgs bundle. The r-differential q induces a subharmonic weight $ \phi_q$ on $K_X \to X$, and the diagonal harmonic metric depends solely on this weight ϕ_q. Previously, I introduced and studied the extension of harmonic metrics associated with an arbitrary subharmonic weight φ. The purpose of considering such a generalization was to explore potential connections between the theory of harmonic metrics on cyclic Higgs bundles and potential theory, including the theory of zero distributions of holomorphic sections of line bundles.
In this presentation, I will explain my ongoing research on the ``statistical-mechanical'' formalism for the theory of harmonic metrics on cyclic Higgs bundles in the generalized sense described above. I will introduce several thermodynamic functions defined using harmonic metrics on cyclic Higgs bundles and provide estimates for these functions. The maximum principle techniques developed by Dai-Li and Li-Mochizuki play a crucial role in deriving these estimates.

15 **********
Your Name：Lucas Henrique Silveira Gomes
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Title: Vaisman Solvmanifolds as a Finite Quotient of Nilmanifolds.

Abstract: A Vaisman structure is a Locally Conformally Kähler metric (LCK for short) such that the Lee form is parallel with respect to the Levi-Civita connection. Bazzoni showed in 2018 that if a nilmanifold admits a Vaisman structure, not necessarily left-invariant, then the associated nilpotent Lie group has to be isomorphic to the one dimensional extension of the 2n+1 dimensional Heinsenberg group. In this talk we show a generalization of this result: every solvmanifold admitting a, not necessarily left-invariant, Vaisman structure has to be a finite quotient of a nilmanifold obtained from the one dimensional extension of the Heisenberg group. As an application we show that there are no Einstein-Weyl LCK structures on solvmanifolds.

16 **********
Your Name：Tadashi Udagawa
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Title: The tt*-equation constructed from the SU(2)k-fusion ring and its DPW description.

Abstract: The tt*-equations were introduced by S. Cecotti and C. Vafa for describing a deformation of supersymmetric conformal field theories. In mathematics, the tt* equations were formulated by B. Dubrovin as a flatness condition on a flat bundle. Dubrovin observed that the tt*-equatons give puriharmonic maps into the symmetric spaces GL(n,R)/O(n). In general, the tt* equation is highly nonlinear and it has been solved in very special cases, but Cecotti and Vafa introduced some examples of solvable tt*- equations such as the sinh-Gordon equation and the tt*-Toda equation. The radial sinh-Gordon equation was solved by B. McCoy, C. Tracy and T. Wu from the viewpoint of p.d.e. theory (1997). The tt*-Toda equation was solved by M. Guest, A. Its and C.-S. Lin from the viewpoint of p.d.e theory (2015) and isomonodromy theory (2015, 2020).

In this talk, we give another examples of solvable tt*-equation due to the idea of Cecotti and Vafa. The consrruction was sketched in the language of conformal field theory and it involves the (SU(2))k-fusion ring, or Verlinde algebra, an object which has a prominat role in conformal field theory. We give the solution constructed directly from a finite number of solutions to the radial sinh-Gordon equation. First, we give a mathematical formulation of the tt*-equation constructed from the (SU(2))k-fusion ring and its solution. Second, we give a holomorphic 1-form corresponding to the solution by using the DPW method (the generalized Weierstrass representation). The DPW method is a way for constructing a harmonic map into the symmetric space from a holomorphic 1-form. Third, we give a natural equivalence relation on representations of SU(2) corresponds to an equally natural notion of gauge equivalence on harmonic maps.

17 **********
Your Name：Shuhei Yonehara
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Title: Actions of cosymplectic groupoids and a reduction theorem.

Abstract: The Mikami–Weinstein reduction theorem is a generalization of the classical Marsden–Weinstein reduction theorem to the case of symplectic groupoid actions. In recent years, the notion of cosymplectic groupoids, which is an odd-dimensional counterpart of the one of symplectic groupoids, has begun to be studied. In this talk, we introduce the notion of cosymplectic groupoid actions on cosymplectic manifolds using the notion of Lagrangian–Legendrean submanifolds of cosymplectic manifolds. Following this, we prove a theorem which is a natural analogue of the Mikami-Weinstein theorem.

18 **********
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Title: Special metrics on crepant resolutions of nodal Calabi-Yau threefolds.

Abstract: Following the conjecture by Reid known as “Reid’s Fantasy”, singular transitions have gained huge interest in the study of Calabi-Yau threefolds, especially conifold transitions. One of the characteristics of the latter is that they are not confined in the Kähler realm, that is, the process can produce manifolds which are not Kähler but still Calabi-Yau (meaning that they still have holomorphically trivial canonical bundle). The same happens if we consider the reverse transition, i.e. if we consider a degenerating family of Kähler Calabi-Yau threefolds with singular fibre carrying only ordinary double points, then its small resolution may not be Kähler. In this joint work with Cristiano Spotti, we show that nevertheless, these small resolutions always carry Chern-Ricci flat balanced metrics in the same balanced class as the singular metric on the base, presenting themselves as a good starting block to produce solutions for the Hull-Strominger system.

19 **********
Your Name：Fumika Mizoguchi
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Title: Nilpotent Lie algebras obtained by quivers and geometric structures.

Abstract: In the study of geometric structures on nilmanifolds, two-step nilpotent Lie algebras obtained by graphs serve as important examples. Recently, additional examples of nilpotent Lie algebras have been constructed from finite quivers without cycles. These Lie algebras can have arbitrarily high degrees of nilpotency, and admit Riemannian Ricci soliton metrics. In this talk, we focus on the two-step nilpotent Lie algebras of this class, and we prove that they can also be obtained by graphs. Using this relationship, we prove that every two-step nilpotent Lie algebra obtained by a finite quiver without cycles admits a pseudo-Riemannian Ricci-flat metric.Moreover, we also classify these nilpotent Lie algebras that admit symplectic structures.

20 **********
Your Name：Hiroaki Nagaya
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Title：A study of bornological proper actions.

Abstract：The studies of proper actions are important to consider the quotient space.
For example, if a Hausdorff group action on a Hausdorff space is proper, the quotient space is also Hausdorff.
Consider a metric space with a continuous isometric action. Then the action is proper if and only if each orbit is closed and every isotropy group is compact. Further, in the setting where a Lie group act on a manifold, Palais proved that every proper action is isometric (Annals of Mathematics, 1961). In this presentation, we study proper actions in the perspective of bornology and coarse geometry. These two fields focus on the concept of ``boundedness''. As the first main result, we introduce the notion of bornological proper actions. It is a generalization of topological proper actions. Additionally, we obtain a characterization of bornological proper actions in the setting of spaces equipped with coarse structures and equi-controlled actions. Finally, we prove that a bounded action is proper only if it is equi-controlled.

21 **********
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Title: An algebraic generalization of association schemes and the indization of a category of the algebras.

Abstract: An association scheme is a generalization of finite homogeneous spaces and is one of the important objects in algebraic combinatorics. An association scheme has an algebra called the Bose--Mesner algebra, which corresponds to Hecke algebra, and harmonic analysis can be developed on association schemes. In this talk, we will give a definition of ``algebras'' with two multiplication structure, having the Bose--Mesner algebra as an example. We will also give a canonical functor from a category of association schemes to the category of monomorphisms of a category of ``algebras''. Furthermore, in the commutative case, we will discuss the indization of the ``algebras''.

22 **********
Your Name：Giuseppe Barbaro
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Title: The pluriclosed flow and the Vaisman condition.

Abstract: The pluriclosed flow has been introduced by Streets and Tian to apply the analytic techniques of geometric flows to the study of complex non-Kahler manifolds. As a matter of fact, it is particularly well-suited for studying the topology and geometry of compact non-Kahler surfaces, with important implications also in their geometrization. Up to now, the long-time behavior of the flow is well understood only in specific cases, in the presence of symmetries or

curvature restrictions. Therefore, it is natural to impose additional conditions to understand the pluriclosed flow solutions better. We thus address the evolution of Vaisman metrics. For compact Vaisman surfaces, we prove that the pluriclosed flow preserves the Vaisman condition if and only if the starting metric has constant scalar curvature. In this case we are reduced to the evolution of locally homogeneous metrics on surfaces, which has been completely described. To prove our result we exploit the fact that the Vaisman structure give rise to a free action of a complex abelian Lie algebra preserving the Hermitian structure. This symmetry is preserved by the flow which can then be studied after a dimensional reduction of the problem.