The 7th Workshop "Complex Geometry and Lie Groups"

May 22 - 26, 2023, Lecce, Italy

Abstract

Daniele Angella: *Plurisigned Hermitian metrics.*

On a compact Hermitian manifold (X, ω), we study the asymptotic behaviour of Monge-Ampère volumes of Hermitian metrics in the $\sqrt{-1}\partial\overline{\partial}$-class of ω, in particular, we ask when they are uniformly bounded and uniformly positive. This motivates the study of plurisigned hermitian metrics.
The talk is a joint collaboration with Vincent Guedj and Chinh H. Lu.

Vestislav Apostolov: *A Calabi type problem in generalized Kähler geometry.*

The notion of a generalized Kahler (GK) structure was introduced in the early 2000's by Hitchin and Gualtieri in order to provide a mathematically rigorous framework of certain nonlinear sigma model theories in physics. Since then, the subject has developed rapidly. It is now realized, thanks to more recent works of Hitchin, Goto, Gualtieri, Bischoff and Zabzine, that GK structures are naturally attached to Kahler manifolds endowed with a holomorphic Poisson structure. Inspired by Calabi's program in Kahler geometry, which aims at finding a "canonical" Kahler metric in a fixed deRham class, I will present in this talk an approach towards a "generalized Kahler" version of Calabi's problem motivated by an infinite dimensional moment map formalism. As an application, we give an essentially complete resolution of the problem in the case of a toric complex Poisson variety. Based on a joint works with J. Streets and Y. Ustinovskiy.

Leonardo Biliotti: *Reduction principles for proper actions.*

Let G be a compact Lie group acting on a compact connected manifold M. Let H be the principal isotropy and let $N(H)$ be the normalizer of H. Let M_r denote the closure of the intersection of the fixed point set M^H with the set of p in M such that the stabilizer G_p is conjugate to H. It is well known that the quotient group $N(H)/H$ acts effectively on M_r and the natural inclusion of M_r into M induces a bijection at orbit space level, $M/G = M_r/N(H)$. In this talk we point out that the same reduction principle holds for proper actions. As an application we show that a reduction principle holds for polar and coisotropic actions. We also briefly discuss coisotropic actions on Hermitian symmetric space of noncompact tyoe. Finally, we completely characterize asystatic actions on almost Kähler manifolds generalizing a result proved by Podestá and Gori (Trans. Group (2006)).

Marco Castrillón López: *Homogeneous structures from the origins to the most recent results.*

Ambrose and Singer gave a very elegant characterization of connected, simply connected and complete homogeneous Riemannian manifolds in terms of a set of geometric equations on a tensor, now a days called homogeneous structure tensor. This seminal work has been extended by many authors to many other Riemannian and semi-Riemannian settings: Kahler, contact, quaternion-Kahler, etc. In this talk we will give a brief exposition of these results as a motivation to the most recent works on the topic in the realm of symplectic or spinorial geometry. Some applications of the results will be also given.

Joana Cirici: *Hypercommutative algebra structures of Calabi-Yau manifolds.*

Motivated by the mirror symmetry program, Barannikov and Kontsevich defined a hypercommutative algebra structure on the cohomology of any compact Calabi-Yau manifold. In fact, they gave a recipe to define such a structure on the cohomology of any Batalin-Vilkovisky algebra satisfying certain conditions. In this talk, I will explain a refinement of this recipe that gives the hypercommutative structure at the cochain level and how, in the case of Calabi-Yau manifolds, one can use Hodge theory to prove results of formality for such algebraic structures. This is joint work with Geoffroy Horel.

Vicente Cortés: *Recent advances in quaternionic Kähler geometry.*

I will show that complete non locally symmetric quaternionic Kähler manifolds with an end of finite volume (and another end of infinite volume) exist in all dimensions divisible by 4. More generally, I will review various constructions which yield quaternionic Kähler manifolds fibering over a line with fibers of finite volume and explain some relations with string theory. The talk is based on the following papers: 1. arXiv:2205.13806; 2. C.-, Röser and Thung, Annales Inst. Fourier (to appear); 3. C.- and Tulli, Commun. Math. Phys. (2022) and 4. work in progress with Tulli.

Eleonora Di Nezza: *Monge-Ampère energies.*

Let X be a compact Kähler manifold. Given a measure μ on X, we look at the complex Monge-Ampère equation $MA(u) = \mu$. In this talk we give a criteria to ensure the existence of a solution in a weighted energy class answering a question raised by Guedj and Zeriahi in 2007. This is a joint work with Darvas and Lu.

Mario Garcia Fernandez: *Vertex algebras from the Hull-Strominger system.*

Motivated by the programme on mirror symmetry for non-Kähler manifols, we construct representations of the $N = 2$ superconformal vertex algebra from solutions of the Hull-Strominger system. For this, we use the Chiral de Rham complex and characterizations of the equations in terms of generalized geometry. Our main results depend on two conditions: firstly, on the vanishing of a bivector field σ canonically associated to a solution of the system, and, secondly, on the connection ∇, one the unknowns of the system, being Hermitian-Yang-Mills. Combined with the results in 2012.01851 (to appear in IMRN), we show that all the known solutions of the Hull-Strominger on non-Kähler threefolds satisfying the second condition have an associated $N = 2$ representation. We also prove a general existence result for these representations on compact complex surfaces. Joint work with Luis Álvarez Cónsul and Andoni de Arriba de la Hera.

Gueo Grantcharov: *Hermitian metrics on compact complex homogeneous spaces.*

Complex manifolds which admit a compact transitive group of biholomorphic transformations were first studied by Wang and are known to admit a canonical fibration over a rational homogeneous space. In this talk we'll consider the existence of special metrics on such spaces like balanced, pluriclosed (or CYT), astheno-Kähler and CYT. Many of the results are based on this canonical fibration. The talk is partly based on a joint works with A. Fino and L. Vezzoni.

Yoshinori Hashimoto: *Hörmander estimates for flat nontrivial line bundles.*

Hörmander's L^2-estimates for the dbar operators on holomorphic line bundles are of fundamental importance in complex analytic geometry, whose conventional proof relies on the positivity of the line bundle. In this talk, we prove the L^2-estimates for the solutions to the dbar equation that hold uniformly for all flat nontrivial line bundles on compact Kähler manifolds, whose main feature is the quantitative description of the blow-up behaviour as the line bundle approaches the trivial one. A key ingredient in the proof is the observation that line bundles with vanishing first Chern classes are topologically trivial and can be identified with the trivial bundle with the "perturbed" dbar operator which we define in terms of coordinates on the Picard variety. This is a joint work with Takayuki Koike.

Nobuhiro Honda: *A construction of 3-dimensional Zoll manifolds via twistor method.*

We use a twistor correspondence between complex analytic objects and differential geometric structures to construct indefinite Einstein-Weyl structures on a 3-manifold. These structures have the special property that all space-like geodesics on them are closed. These geometric structures may be regarded as a generalization of the deSitter space, which is the indefinite space form with positive curvature. The (mini) twistor spaces we use to obtain these Einstein-Weyl spaces are projective algebraic surfaces that can be explicitly constructed, and the proof mostly relies on algebraic geometry.

Jun Muk Hwang: *Formal principle with convergence for rational curves of Goursat type.*

We propose a conjecture that a general member of a bracket-generating family of rational curves in a complex manifold satisfies the formal principle with convergence, namely, any formal equivalence between such curves is convergent. If the normal bundles of the rational curves are positive, the conjecture follows from the results of Commichau-Grauert and Hirschowitz. We prove the conjecture for the case when the normal bundles are furthest from positive vector bundles among bracket-generating families, namely, when the families of rational curves are of Goursat type. As an example, we see that a general line on a smooth cubic fourfold satisfies the formal principle with convergence.

Hisashi Kasuya: *Higgs bundles and uniformizations of compact Sasakian manifolds.*

In the joint works with I. Biswas(TIFR), we prove the non-abelian Hodge correspondence on compact Sasakian manifolds as an odd dimensional counterpart of the Corlette-Simpson theorem on compact Kähler manifolds. By using this, in the joint works with N. Miyatake(Tohoku), we give a uniformization theorem of compact Sasakian manifolds as a Sasakian version of the Hitchin-Simpson uniformization theory and also as an higher dimensional extension of Belgun's classification of 3-dimensional compact Sasakian manifolds.

Adela Latorre: *Nilmanifolds with a special type of complex structures.*

A nilmanifold $M = \Gamma\backslash G$ is a quotient of a connected, simply connected, nilpotent Lie group G by a cocompact discrete subgroup Γ. An invariant complex structure on M is a complex structure J defined on the Lie algebra $\mathfrak{g}$ of G, namely, an endomorphism $J : \mathfrak{g} \to \mathfrak{g}$ satisfying $J^2 = -Id$ and $N_J \equiv 0$, being N_J the Nijenhuis tensor of J. If we denote $\mathcal{Z}(\mathfrak{g})$ the center of $\mathfrak{g}$, two cases arise: either $\mathcal{Z}(\mathfrak{g})$ contains a J-invariant ideal of $\mathfrak{g}$ or $\mathcal{Z}(\mathfrak{g}) \cap J\big(\mathcal{Z}(\mathfrak{g})\big) = \{0\}$. When the latter holds, the complex structure J is called strongly non-nilpotent or SnN, for short. In this talk, we will focus on this specific type of invariant complex structures on nilmanifolds, providing their classification in real dimension eight. This will allow us to study the behaviour of their Frölicher spectral sequence in relation to some special Hermitian metrics.

Andrea Loi: *Rigidity properties of holomorphic isometries into homogeneous Kähler manifolds.*

The aim of the talk is to provide an overview of some recent rigidity phenomena on holomorphic isometries into homogeneous Kähler manifolds which extend some classical results in the complex space forms case. In particular, we focus on the unicity of a given holomorphic isometry $f : M \to S$ between a Kähler manifold (M, g) and a homogeneous Kähler manifold (S, h) and on the obstruction for the existence of such map when the metric g is canonical (e.g. csck, extremal, Kähler-Einstein or a Kähler Ricci soliton). Moreover, we address and discuss the problem of characterizing those homogeneous Kähler manifolds which are relatives, i.e. which share a common non trivial Kähler submanifold.

Shin-ichi Matsumura: *On projective manifolds with semi-positive holomorphic sectional curvature.*

In this talk, I would like to discuss the geometry of projective manifolds with semi-positive holomorphic sectional curvature after I review several classical results for holomorphic bisectional curvature. I first show that, if X has positive holomorphic sectional curvature, then X is rationally connected, that is, any two points can be connected by a rational curve (the image of CP^1 by a holomorphic map). This result proves a generalization of Yau's conjecture (which was proved by Heier-Wong and Yang). Moreover, I show that if X has semi-positive holomorphic sectional curvature, X admits a locally trivial morphism from X to Y such that the fiber F is rationally connected and the base Y is a flat manifold. The proof depends on the theory of holomorphic foliations, MRC fibrations, and singular Hermitian metrics.

Takeo Ohsawa: *Geometry of analytic continuations–history, survey and report.*

In function theory of several complex variables, various geometric questions arise in connection to the analytic continuation. Beginning from the state of art around 1954, solutions of the Levi problem on complex manifold will be reviewed at first up to Takayama's result in 1998. Then the activity of extending the results by the L^2 method in these decades will be reviewed. The method is by exploiting the finite-dimensionality of certain L^2 dbar-cohomology groups to prove that a Hermitian holomorphic line bundle L over a complex manifold M is bimeromorphically equivalent to an ample bundle when it is restricted to a bounded locally pseudoconvex domain $\Omega \Subset M$ under certain geometric conditions on L and $\partial\Omega$.

Duong H. Phong: *Symplectic geometric flows and almost-complex structures.*

The equations of unified string theories have led to several new interesting geometric flows. One of which is the Type IIA flow, which is a weakly parabolic flow on 6d symplectic manifolds whose underlying geometry turns out to be SU(3) holonomy, but for a *projected* Levi-Civita connection with respect to an almost-complex structure. We discuss this flow and many open problems on related flows in symplectic geometry. This is joint work with Teng Fei, Sebastien Picard, and Xiangwen Zhang.

Jonas Stelzig: *Pluripotential homotopy theory.*

Sullivan has asked whether one can build models (in the sense of rational homotopy theory) for the differential graded algebra of differential forms on complex manifolds that are compatible with the bigrading and the real structure. I will answer this question positively both in a conceptual and a computational sense, by exhibiting a model structure on the category of bigraded, bidifferential algebras and by developing a theory of bigraded minimal models. The notion of quasi-isomorphism used is very strong and allows to track 'pluripotentials', i.e. solutions to equations of the form x=ddbar y. As applications, one obtains refinements of the (higher) homotopy groups which are sensitive to the complex structure. Further, certain higher operations involving pluripotentials, like the Bott-Chern-Aeppli Massey products defined by Angella, Tomassini and Tardini, find a natural place in this theory as obstructions to a version of formality.

Adriano Tomassini: *On spaces of harmonic forms and cohomology of almost complex manifolds.*

Let (M, J, g, ω) be a compact almost Hermitian manifold. Then the *Dolbeault*, *Bott-Chern* and *Aeppli* Laplacians are naturally defined on (M, J, g, ω). We will report on some recent results on the study of the kernels of such operators. Furhermore, we will define the *Bott-Chern cohomology* of an almost complex manifold (M, J). The results have been obtained in some papers joint with A. Fino, T. Holt, R. Piovani, L. Sillari, N. Tardini.

Luigi Vezzoni: *The Calabi-Yau problem on Hypercomplex manifolds.*

The talk focuses on special metrics that are compatible with hypercomplex structures. In particular, we will consider the hyperKähler with torsion metrics (HKT) introduced by Howe and Papadopoulos in 1996. HKT manifolds represent the hypercomplex counterpart of Kähler manifolds in many ways, and in this context balanced HKT metrics play the same role of Calabi-Yau metrics in Kähler geometry. In 2010 Alesker and Verbitsky introduced a Calabi-Yau type problem in HKT geometry, stating a conjecture that is still open. In the talk, we will discuss both some partial advances in the study of the conjecture on some explicit examples and an approach to investigate it via a parabolic flow.