The 6th Workshop "Complex Geometry and Lie Groups"

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ONLINE

Abstract

Hisashi Kasuya: *Higgs bundles and flat connections over compact Sasakian manifolds and their applications.*

On a compact Kahler manifold, there is an equivalence between semi-simple flat vector bundles and polystable Higgs bundles with vanishing Chern classes via harmonic metrics (Corlette, Simpson). We give an analogue of this equivalence on compact Sasakian manifolds (odd-dimensional counterpart to Kahler manifolds). We prove that on a compact Sasakian manifold, there is an equivalence between semi-simple flat bundles and polystable basic Higgs bundles with vanishing basic Chern classes via harmonic metrics (Joint work with I. Biswas). Applying this equivalence, we prove some topological properties of compact Sasakian manifolds.

Luigi Vezzoni: *Geometric flows of Hermitian metrics on Lie groups.*

The talk focuses on geometric flows of Hermitian metrics on non-Kähler manifolds, paying particular attention to the family of Hermitian curvature flows introduced by Streets and Tian. It will be shown that, under suitable assumptions, an Hermitian Curvature flow starting from a left-invariant Hermitian metric on a Lie group has a long time solution converging to a soliton, up to renormalization. The study of solitons and static solutions of geometric flows on Lie groups will be also addressed. The results have been mainly obtained in collaboration with Nicola Enrietti, Anna Fino, Ramiro Lafuente and Mattia Pujia.

Fabrizio Catanese: *Automorphism groups of compact Kaehler manifolds and surfaces.*

I shall try, before dealing with recent results and work in progress together with Wenfei Liu, concerning several subgroups of topologically trivial automorphisms, to outline some general results on the topic (time permitting).

Eugenia Loiudice: *How to construct f-K-contact and S-manifolds.*

Slide is available in SLACK

Ryosuke Takahashi: *Some geometric flow approaches for deformed Hermitian-Yang-Mills equation.*

On SYZ mirror symmetry, a deformed Hermitian-Yang-Mills (dHYM) metric is a fiber metric on a holomorphic line bundle, which is the mirror object to a special Lagrangian section of the dual torus fibration. As a parabolic analogue, Jacob-Yau'17 introduced the Line Bundle Mean Curvature Flow (LBMCF) which is the mirror of the Lagrangian Mean Curvature Flow (LMCF) for graphs. The LBMCF has many similar properties to the LMCF. However, the long time existence and convergence of these flows is a subtle matter. For example, Neves'13 showed that the LMCF forms finite time singularities even if there exists a special Lagrangian. In this talk, we explore some geometric flow approaches from the following different view points:

(A) On Kähler surfaces, it is known that the existence of dHYM metrics is equivalent to a certain positivity condition for a cohomology class. We relax this positivity to semipositivity and study how the LBMCF blows up. (B) Recently, Collins-Yau'18 discovered a GIT/moment map interpretation for dHYM metrics based on the earlier works of Solomon'13 and Thomas'01 in the mirror side. Motivated by this, we introduce a new geometric flow which is designed to deform a given metric to a dHYM one. Then we show that this new flow potentially has more global existence and convergence properties compared to the LBMCF.

Alexandra Otiman: *Special non-Kähler metrics on solvmanifolds*

We discuss old and new results about the existence of special Hermitian metrics (locally conformally Kähler, balanced, pluriclosed) on complex nilmanifolds and on Oeljeklaus-Toma manifolds. This latter class represents a generalization of Inoue-Bombieri surfaces in arbitrary complex dimension and its construction, based on algebraic number theory, will allow us to give a numerical interpretation for the existence of several special type Hermitian metrics.

Dan Popovici: *Higher-Page Hodge Theory of Compact Complex Manifolds.*

This is joint work with Jonas Stelzig (Munich) and Luis Ugarte (Zaragoza). The main thrust of this work is to extend some basic results in Hodge Theory to the higher pages of the Frölicher spectral sequence. For an arbitrary nonnegative integer r, we introduce the class of page-r-$\partial\bar\partial$-manifolds by requiring the analogue of the Hodge decomposition to hold on a compact complex manifold X when the usual Dolbeault cohomology groups $H^{p,\,q}_{\bar\partial}(X)$ are replaced by the spaces $E^{p,\,q}_{r+1}(X)$ featuring on the $(r+1)$-st page of the Frölicher spectral sequence of X. The class of page-r-$\partial\bar\partial$-manifolds increases as r increases and coincides with the usual class of $\partial\bar\partial$-manifolds when $r = 0$. We investigate various properties of these manifolds and show that they are analogous to those of $\partial\bar\partial$-manifolds with some noteworthy exceptions. We also point out a number of examples. We also introduce higher-page analogues of the Bott-Chern and Aeppli cohomologies and highlight their relations to the new class of manifolds.

Daniele Angella: *Locally conformally Kähler threefolds of algebraic dimension 2.*

In this talk, we describe the structure of complex threefolds with algebraic dimension 2 in the case when they admit a metric that is locally conformal to Kähler metrics. In particular, we show that, under mild assumptions, every such manifold is essentially an elliptic fibration over a compact projective surface with isomorphic fibers. The talk is a joint collaboration with Maurizio Parton and Victor Vuletescu.

Kazushi Ueda: *Homological mirror symmetry for affine K3 surfaces.*

We discuss homological mirror symmetry for affine K3 surfaces obtained as the complements of smooth ample divisors in K3 surfaces. In general, we conjecture that the mirror is a type III degeneration of K3 surfaces whose index and t-invariant are given by the primitivity index and the degree of the ample divisor. The case when the t-invariant is divisible by 4 is related to that of an abelian surface via a double cover. This is a joint work with Yanki Lekili.

Oliver Goertsches: *Hamiltonian non-Kähler actions in low dimensions.*

We classify 3-valent GKM fiber bundles over n-gons, show that they are all realized as the projectivization of equivariant complex rank 2 vector bundles over quasitoric 4-manifolds, and investigate the existence of invariant (stable) almost complex, symplectic, and Kähler structures on the total space. In this way we obtain infinitely many new examples of Hamiltonian non-Kähler actions in dimension 6 with prescribed shape of the x-ray, in particular with prescribed number of fixed points. We extend our methods to give interesting examples of torus actions in dimension 8 that answer a natural cohomological rigidity question. This is joint work with Panagiotis Konstantis and Leopold Zoller.

Francis E. A. Johnson: *Fibering by locally symmetric spaces.*

Note is available in SLACK

Kotaro Kawai: *Deformation theory of deformed Donaldson–Thomas connections.*

The deformed Donaldson-Thomas (dDT) connection is a Hermitian connection of a Hermitian line bundle over a G_2-manifold satisfying certain nonlinear PDEs. This is considered to be the mirror of a (co)associative submanifold via mirror symmetry. As the name indicates, the dDT connection can also be considered as an analogue of the Donaldson–Thomas connection (G_2-instanton). (Further, it is considered to be an analogue of the deformed Hermitian Yang-Mills (dHYM) connection over a Kähler manifold.) In this talk, we show that dDT connections have similar properties to associative submanfiolds and G_2-instantons. That is, the moduli space is an orientable smooth finite dimensional manifold for generic G_2-structures. This might shed light on the enumerative problem in G_2-geometry. This is joint work with Hikaru Yamamoto.

Kengo Hirachi: *Local and global invariants of CR geometry.*

CR geometry is modeled on the homogeneous space $SU(n, 1)/P$, where P is a parabolic subgroup.Invariant theoretic study of CR geometry was initialized by Fefferman and is now called Parabolic invariant theory. In this talk, I will survey the developments in the constructions of various CR invariants (Q and Q-prime curvatures and their further generalizations by T. Marugame) from the point of view of the groups $SU(n, 1)$ and P.

Kazuyuki Hasegawa: *The quaternionic/hypercomplex-correspondence.*

Given a quaternionic manifold M with a certain U(1)-symmetry, we construct a hypercomplex manifold M' of the same dimension. This construction generalizes the quaternionic Kähler/hyper-Kähler-correspondence. As an example of this construction, we obtain a compact homogeneous hypercomplex manifold which does not admit any hyper-Kähler structure. Therefore our construction is a proper generalization of the quaternionic Kähler/hyper-Kähler-correspondence. This talk is based on a joint work with V. Cortés (arXiv:1904.06056).

Alberto Raffero: *Symplectic half-flat manifolds with large symmetry group.*

A six-dimensional almost Kähler manifold is said to be symplectic half-flat if its frame bundle admits an SU(3)-reduction by means of a normalized complex volume form with closed real part. In this talk, I will focus on symplectic half-flat manifolds having a large symmetry group, discussing existence and classification results both in the compact and in the non-compact setting. This is joint work with F. Podestà.

Federico A. Rossi: *Diagram involutions and homogeneous Ricci-flat metrics.*

The construction of metrics satisfying the Einstein equation ric $= \lambda$ g is a classical problem both in Riemannian and pseudo-Riemannian geometry. In particular, homogeneous Ricci-flat indefinite metrics can be non flat. In this talk we will introduce a systematic construction of Ricci-flat indefinite metrics on a large class of homogeneous spaces: nice nilpotent Lie groups. Since we deal with left-invariant metric, we focuson scalar products on the Lie algebras of those nilpotent groups, called nice nilpotent Lie algebras. It turns out that those Lie algebras, which play an important role in the study of Ricci flow and in the construction of nilsolitons, can be constructed and classified explicitly. Moreover they have a combinatorial nature and a special directed graphs can be attached to them. The construction of Ricci-flat metrics is based on combinatorial tools: special diagram involutions called "arrow-breaking". We will discuss many examples that can admits this construction (two-step nilpotent Lie group attached to a graph, nilmanifolds associated to parabolic nilradicals, filiform nilpotent Lie groups), showing that families of nonisometric Ricci-flat metrics can arise and that most of these metrics are shown not to be flat. Time permitting, I will show interesting links between diagram involutions and the construction of ad-invariant metrics on nice nilpotent Lie groups. This is a joint work with D. Conti and V. del Barco.

Nobuhiro Honda: *Segre surfaces and minitwistor spaces with genus one.*

A complex surface which has a smooth rational curve whose self-intersection number is two is called a minitwistor space. These rational curves constitute a 3-dimensional family, and it is known that the parameter space of this family is naturally equipped with a special geometric structure, called an Einstein-Weyl structure (Hitchin 1982). About 10 years ago, in collaboration with F. Nakata, we showed that the same result as Hitchin holds even when we allow a correct number of ordinary double points to the rational curves in a complex surface. It seems natural to call the number of ordinary double points the genus of the minitwistor space. In this talk, we present our recent result that minitwistor spaces with genus one are exactly quartic surfaces in $\mathbf{CP}^4$ which are called Segre surfaces, and the Einstein-Weyl spaces corresponding to Segre surfaces are Zariski open subsets of the dual varieties of the Segre surfaces. Details can be found in arXiv:2009.05242.

Leonardo Biliotti: *Satake-Furstenberg compactifications and gradient map.*

Let G be a real semisimple Lie group with finite center and let K be a maximal compact subgroup of G. The G action on the projective space of V admits a natural G-gradient map and G has a unique closed orbit which is a K orbit. The image of the G-gradient map restricted to the unique closed orbit of G is a K orbit in p and Its convex hull is denoted by E. We prove that, up to equivalence, the set of irreducible representations of parabolic subgroups of G induced by R are completely determined by the facial structure of E. Moreover, any parabolic subgroup of G admits a unique closed orbit which is well-adapted to the unique closed orbit of G and the G-gradient map. These results are new also in the complex reductive case. The connection between E and R provides a geometrical description of the Satake compactifications of G/K without root data. In this context the properties of the Bourguignon-Li-Yau map are also investigated.

Vicente Cortés: *Generalized connections, integrability, and T-duality.*

We characterize the integrability of various structures on Courant algebroids in terms of torsion-free generalized connections. The applications include generalized Kähler and generalized hyper-Kähler structures as particular examples. We do also give a spinorial characterization in the case of regular Courant algebroids. This is based on the theory of Dirac generating operators, for which we give a new approach based on the geometric data encoding a regular Courant algebroid. Finally, we develop a theory of T-duality for transitive Courant algebroids. The talk is based on joint work with Liana David, see arXiv:1905.01977 and arXiv:2101.07184.

Michela Zedda: *Stability with respect to actions of real reductive Lie groups.*

The action of a real reductive Lie group G acting on a real compact submanifold M of a Kaehler manifold, induces in a natural way a continuous action of G on measures of M. The aim of this talk is to describe stability theory in this setting. The talk is based on a joint work with Leonardo Biliotti (2017).

Yuji Odaka: *Collapsing hyperKahler manifolds and Satake compactification of adjoint type.*

This talk is mainly based on a joint work with Y.Oshima (arXiv:1810.07685), and possibly some related papers. We apply one of Satake compactifications, which we call adjoint type being different from Baily-Borel, to moduli of K3 surfaces which is known to have locally symmetric space structure. Then, we let its boundary parametrize metric spaces with additional structures and partially prove that they are nothing but the limit of hyperKahler metrics. If time permits, we may also discuss related topics.

Adriano Tomassini: Let J be an almost complex structure on a $2n$-dimensional manifold M. Then J acts in a natural way as an involution on the bundle of 2-forms $\Lambda^2(M)$, so that

$$\mathcal{A}^2(M) = \mathcal{A}^+_J(M) \oplus \mathcal{A}^-_J(M),$$

where $\mathcal{A}^2(M)$ is the space of smooth sections of $\Lambda^2(M)$, $\mathcal{A}^+_J(M)$ and $\mathcal{A}^-_J(M)$ denote the space of J-invariant and J-anti-invariant forms, respectively. Therefore, it is natural to consider the cohomology subgroups $H^+_J(M)$, $H^-_J(M)$ of $H^2_{dR}(M;R)$ whose elements are de Rham classes represented by J-invariant, J-anti-invariant forms respectively.
We will present some recent results obtained in a joint paper with Richard Hind, concerning the anti-invariant cohomology of almost complex manifolds.

Nicolina Istrati: *Toric Kato manifolds.*

Kato manifolds are compact complex manifolds which admit a global spherical shell, i.e. a well-embedded sphere which does not disconnect the manifold. They have infinite cyclic fundamental group, and so are non-Kählerian, but many of them admit locally conformally Kähler (lcK) metrics. Moreover, in complex dimension 2, they play an important role in the classification of class VII surfaces. In this talk, I will first give an introduction to Kato manifolds and a motivation of their study coming from lcK geometry. Then, I will describe how one can construct a special class of them, using the language of toric algebraic geometry. Using the toric description, I will then explain how one can compute some of their analytic invariants, or study some of their complex submanifolds. Finally, I will say a few words about the existence of distinguished Hermitian metrics on Kato manifolds. This is based on joint work with A. Otiman, M. Pontecorvo and M. Ruggiero.

Zakarias Sjostrom Dyrefelt: *Constant scalar curvature and Kähler manifolds with nef canonical bundle.*

Using recent results of Chen-Cheng and Weinkove we prove existence of constant scalar curvature metrics on Kähler manifolds with nef canonical bundle, extending the classical result of Aubin and Yau (and later work of Jian-Shi-Song) to smooth minimal models. The tools used are from the variational approach in Kähler geometry. Some related results on Donaldson's J-equation are also discussed.