

## ABSTRACTS- PLENARY SPEAKERS

**Takao Akahori** – *Partially integrable almost CR structures.*

Abstract: Let  $(M, D)$  be a compact contact manifold with  $\dim_{\mathbb{R}} M = 2n - 1 \geq 5$ . This means that:  $M$  is a  $C^\infty$  differential manifold with  $\dim_{\mathbb{R}} M = 2n - 1 \geq 5$ . And  $D$  is a subbundle of the tangent bundle  $TM$  which satisfying ; there is a real one form  $\theta$  such that  $D = \{X : X \in TM, \theta(X) = 0\}$ , and  $\theta \wedge \wedge^{n-1}(d\theta) \neq 0$  at every point of  $p$  of  $M$ . Here, we assume that our  $D$  admits an almost CR structure. Rather recently, several topologists are discussing on the relation between contact structures and "stein spaces". Here, we treat a special kind of contact structures, that is to say, a partially integrable almost CR structure. Inspired by the Matsumoto's work (see [M]), we discuss a partially integrable almost CR structure and from the point of view of deformation theory of CR structures. And we show that; even though lack of integrability, several results hold. This work is in progress.

References:

[A1] T. Akahori, *The new estimate for the subbundles  $E_j$  and its application to the deformation of the boundaries of strongly pseudo convex domains*, 63(1981), pp311-334, *Inventiones Mathematicae*.

[A2] T. Akahori, *On the partially integrable almost CR structures*, preprint.

[AGL] Akahori-Garfield-Lee, T. Akahori, P. M. Garfield, and J. M. Lee, *Deformation theory of five-dimensional CR structures and the Rumin complex*, 50(2002), 517-549, *Michigan Mathematical Journal*.

[M] S. Matsumoto, *GJMS Operators, Q-curvature, and obstruction tensor of partially integrable CR manifolds*, arXiv:1402.4110v4[math.DG] 2Mar2015.

**Roger Bielawski** – *Coadjoint orbits, slices, and hyperkaehler metrics.*

Abstract: I'll review the "hyperkaehler slice construction", which starts with a (complete) hyperkaehler manifold with a tri-Hamiltonian action of a compact semisimple Lie group and produces a (complete) hyperkaehler metric on the inverse image (under the complex moment map) of a Slodowy slice. Many well-known examples of complete hyperkaehler manifolds arise this way, and I will describe several interesting new examples. I will also discuss certain common features of manifolds arising from this construction.

**Gueo Grantcharov** – *Algebraic dimension of complex nilmanifolds*

Abstract: This is a joint project with A. Fino and M. Verbitsky. Let  $a(M)$  be the algebraic dimension of a complex manifold  $M$  and  $h(M)$  be the dimension of its space of holomorphic differentials. We show that, If  $M$

is a compact complex nilmanifold, then  $a(M) \leq h(M)$ . We use it to determine  $a(M)$  when  $M$  is 3-dimensional and also mention a relation with the kähler rank of  $M$ .

**Marco Gualtieri** – *Holomorphic Poisson geometry and generalized Kahler structures.*

Abstract: I will review some recent developments in the study of holomorphic Poisson structures and explain how these can be used to construct new examples of generalized Kahler structures. Also, I will explain a new method for expressing generalized Kahler structures purely in terms of Poisson geometry.

**Hisashi Kasuya** – *Extensions of Nomizu's Theorem.*

Abstract: Solvmanifolds (resp. nilmanifolds) are quotients of simply connected solvable (resp. nilpotent) Lie groups by cocompact discrete subgroups. In 1950s, K. Nomizu proved that the de Rham cohomology of nilmanifolds are computed by the invariant differential forms. This theorem was generalized for  $\mathbb{O}$ -special solvmanifolds by A. Hattori and G. D. Mostow. But, we can not obtain a "simple" generalization of Nomizu's Theorem for general solvmanifolds. In this talk, we give techniques of computations of the de Rham cohomology of general solvmanifolds and we give simplicial rational extended Nomizu's Theorem.

**Ryoichi Kobayashi** – *Free Fuchsian groups in classical minimal surface theory.*

Abstract: The purpose of this talk is to reveal the prominent role of the free Fuchsian groups in the classical minimal surface theory.

**Claude Le Brun** – *Mass in Kähler Geometry.*

Abstract: Given a complete Riemannian manifold that looks enough like Euclidean space at infinity, physicists have defined a quantity called the  $\mathbb{O}$ -mass which measures the asymptotic deviation of the geometry from the Euclidean model. In this lecture, I will explain a simple formula, discovered in joint work with Hajo Hein, for the mass of any asymptotically locally Euclidean (ALE) Kähler manifold. For ALE scalar-flat Kähler manifolds, the mass turns out to be a topological invariant, depending only on the underlying smooth manifold, the first Chern class of the complex structure, and the Kähler class of the metric. When the metric is actually AE (asymptotically Euclidean), this formula

not only implies a positive mass theorem for Kähler metrics, but also yields a Penrose-type inequality for the mass.

**Claude Le Brun** – *The Einstein-Maxwell Equations & Conformally Kähler Geometry.*

Abstract: Any constant-scalar-curvature Kähler (cscK) metric on a complex surface may be viewed as a solution of the Einstein-Maxwell equations, and this allows one to produce solutions of these equations on any 4-manifold that arises as a compact complex surface with even first Betti number. However, not all solutions of the Einstein-Maxwell equations on such manifolds arise in this way. In this lecture, I will describe a construction of new compact examples that are Hermitian, but not Kähler.

**Katrin Leschke** – *Quaternionic Holomorphic Geometry.*

Abstract: In my talk, I will give a short introduction to Quaternionic Holomorphic Geometry: conformal maps into 3-space can be used as an analogue for complex holomorphic functions. As an example of the theory I will discuss the Darboux transformation of minimal surfaces. Recent results are joint work with K. Moriya (Tsukuba).

**Toshiki Mabuchi** – *The Donaldson-Tian-Yau Conjecture for general polarizations.*

Abstract: The Donaldson-Tian-Yau Conjecture for Kähler-Einstein cases was solved affirmatively by Chen-Donaldson-Sun and Tian. However for general polarizations, this conjecture is still open. In this talk, we discuss our recent works in progress related to this general problem.

**Massimiliano Pontecorvo** – *Bi-Hermitian metrics on Kato surfaces.*

Abstract: A bi-Hermitian surface is a real-four manifolds with two different complex structures which are Hermitian with respect to a fixed Riemannian metric and induce the same orientation. We will report on some existence results as well as obstructions for such metrics on non-Kähler surfaces.

**Sönke Rollenske** – *Dolbeault cohomology of nilmanifolds with left-invariant complex structure.*

Abstract: I will discuss old and new results to what extent the Dolbeault cohomology of nilmanifolds with left-invariant complex structure can be computed as Lie- Algebra cohomology.

**Yusuke Sakane** – *Einstein metrics on compact homogeneous spaces.*

Abstract: In this talk we describe how we can construct homogeneous Einstein metrics on compact homogeneous manifolds by focusing the

cases of Stiefel manifolds and compact simple Lie groups. For Stiefel manifolds we know there exist at least two homogeneous Einstein metrics by a result of Jensen. So our problem is how we can construct more homogeneous Einstein metrics. For a compact semisimple Lie group, in 1979 D'Atri and Ziller started a study of homogeneous Einstein metrics on compact simple Lie groups and constructed many naturally reductive metrics by giving a characterization of naturally reductive metrics among left-invariant metrics. They also asked whether  $G$  admits a non-naturally reductive Einstein metric. We give a method to construct non-naturally reductive Einstein metrics on compact simple Lie groups. The talk is based on joint works with Andreas Arvanitoyeorgos, Ioannis Chrysikos, Kunihiko Mori and Marina Statha.

**Jeff Streets** – *Generalized Kähler-Ricci flow and nondegenerate generalized Kähler surfaces.*

Abstract: Generalized Kähler structures are rich geometric objects combining aspects of complex and symplectic geometry. A special type of this structure was constructed by Joyce via deformation away from hyperKähler structures. We show that this is the only way to construct such structures in dimension 4 by a geometric flow method. In particular we prove long time existence and convergence of the generalized Kähler-Ricci flow to a weak hyperKähler structure.

**Luigi Vezzoni** – *The Calabi-Yau problem in torus bundles and generalized Monge-Ampère equations.*

Abstract: The Calabi-Yau equation is a PDEs system whose study goes back to the celebrated Calabi conjecture. Recently, Donaldson has described how the equation could be generalized in a natural way to the setting of 2-forms on 4-manifolds. Donaldson's program, if carried out, would lead to many new and important results in symplectic geometry. Given a 4-dimensional compact symplectic manifold  $(M, \Omega)$  together an  $\Omega$ -compatible almost-complex structure  $J$ , the Calabi-Yau equation consists in

$$(\Omega + d\alpha)^2 = e^F \Omega^2, \quad Jd\alpha = d\alpha$$

where  $F \in C^\infty(M)$  is given and  $\alpha$  is a unknown 1-form. In contrast to the Kähler case, it is not known if the equation in the almost-complex setting has always a solution.

The talk focuses on the study of the Calabi-Yau equation in torus fibrations over a torus, when the initial datum  $F$  is invariant by the action of the fiber. It will be showed that in this case the equation reduces to a generalized Monge-Ampère equation on the basis having always a solution.

The last part of the talk will be about some recent advances on the problem in the Kodaira-Thurston manifold.

References:

E. Buzano, A. Fino and L. Vezzoni, The Calabi-Yau equation for  $T^2$ -bundles over a torus: the non-Lagrangian case, *Rend. Semin. Mat. Univ. Politec. Torino* **69** (2011), no. 3, 281–298.

E. Buzano, A. Fino and L. Vezzoni, The Calabi-Yau equation on the Kodaira-Thurston manifold, viewed as  $S^1$ -bundle over a 3-torus, *J. Differential Geom.* **101** (2015), no. 2, 175–195.

A. Fino, Y.Y. Li, S. Salamon and L. Vezzoni, The Calabi-Yau equation on 4-manifolds over 2-tori, *Trans. Amer. Math. Soc.* **365** (2013), no. 3, 1551–1575.

## ABSTRACTS- RESEARCH SPEAKERS

**Masanori Adachi** – *Curvature restrictions for Levi-flat real hypersurfaces in complex projective planes.*

Abstract: We study curvature restrictions of Levi-flat real hypersurfaces in complex projective planes, whose existence is in question. We focus on its totally real Ricci curvature, the Ricci curvature of the real hypersurface in the direction of the Reeb vector field, and show that it cannot be greater than  $-4$  along a Levi-flat real hypersurface. We rely on a finiteness theorem for the space of square integrable holomorphic 2-forms on the complement of the Levi-flat real hypersurface, where the curvature plays the role of the size of the infinitesimal holonomy of its Levi foliation. The talk is based on joint work with J. Brinkschulte.

**Daniele Angella** – *Special Hermitian metrics up to conformal changes on non-Kähler manifolds.*

Abstract: We study problems related to the existence of special metric structures on complex non-Kähler manifolds. In particular, we consider metrics up to conformal changes. We study an analogue of the Yamabe problem in the Hermitian setting (joint work with Simone Calamai and Cristiano Spotti). We also consider the case of local conformal changes.

**Leonardo Biliotti** – *Stability of measures on Kähler manifolds.*

Abstract: Let  $M$  be a Kähler manifold and let  $K$  be a compact group that acts on  $M$  in a Hamiltonian fashion. We will study the action of  $K$  and its complexification on the space of probability measures on  $M$ . First of all we identify an abstract setting for the momentum mapping and give numerical criteria for stability, semi-stability and polystability. Next we apply this setting to the action of the complexification of  $K$  on measures. We get various stability criteria for measures on Kähler manifolds. The same circle of ideas gives a very general surjectivity result for a map originally studied by Hersch and Bourguignon-Li-Yau.

**Xin Dong** – *Boundary asymptotics of the relative Bergman kernel metric for hyperelliptic curves and Jacobians.* Abstract: The Bergman kernel on each complex manifold is a canonical volume-form determined by the complex structure, and we study the variation of Bergman kernels at degeneration. For a holomorphic family of hyperelliptic curves we obtain asymptotic formulas with explicit coefficients of the relative Bergman kernel metric near singularities, which is different from the elliptic curve case where hyperbolic growth exists at the node. I will also discuss general curves and their Jacobians, before ending with Bergman kernels' lower bounds.

**Kenta Hayano** – *Topology of holomorphic Lefschetz pencils on the four-torus.*

Abstract: For a Lefschetz pencil  $f$  on a smooth four-manifold, we can define two basic invariants using the closure  $F$  of a regular fiber of  $f$ ; the genus of  $f$  is defined to be the genus of  $F$ , and the divisibility of  $f$  is the maximum integer by

which we can divide the integral homology class represented by  $F$ . In this talk, we will prove that the (smooth) isomorphism class of a holomorphic Lefschetz pencil on the four-torus is uniquely determined by its genus and divisibility. The proof relies on the theory of moduli spaces of polarized abelian surfaces. This is a joint work with Noriyuki Hamada (The University of Tokyo).

**Hassan Jolany** – *Logarithmic Weil-Petersson metric on moduli space of log Calabi-Yau varieties.*

**Casey Kelleher** – *Singularity Formation of the Yang-Mills Flow .*

Abstract: We explore the structure of the singularities of Yang-Mills flow in dimensions  $n \geq 4$ . First we derive a description of the singular set in terms of concentration for a localized entropy quantity, which leads to an estimate of its Hausdorff dimension. We develop a theory of tangent measures for the flow at such singular points, which leads to a stratification of the singular set. By a refined blowup analysis we obtain Yang-Mills connections or solitons as blowup limits at any point in the singular set. This is joint work with Jeffrey Streets.

**Shinichiroh Matsuo** – *The twisted Seiberg-Witten equations and the Yamabe invariant.*

Abstract: We compute the Yamabe invariants for a new infinite class of closed 4-dimensional manifolds by using a twisted version of the Seiberg-Witten equations. The key ingredient is a new non-vanishing result for the twisted equations. This talk is based on a joint work with N. Nakamura and M. Ishida.

**Katsuhito Moriya** – *Twistor lifts and factorization for conformal maps of a surface.*

Abstract: Conformal maps from a Riemann surface to the four-dimensional Euclidean space are studied by twistor lifts and a quaternionic holomorphic structure. We explain a relation between these objects and define a factorization of the differential of a conformal map. We apply the factorization to constrained Willmore surfaces and minimal surfaces. Then we characterize constrained Willmore surfaces by canonical lifts. We give an upper bound of the area of a minimal surface around a branch point. This is joint work with Kazuyuki Hasegawa.

**Shinnosuke Okawa** – *Compact moduli of marked noncommutative del Pezzo surfaces.*

Abstract: It is known that a smooth projective variety over a field can be reconstructed from the abelian category of coherent sheaves on it. On the other hand, not all deformations of the category (=noncommutative deformations) come from deformations of the variety. In this sense, usual deformations can be regarded as special NC deformations. NC deformations have some intriguing

correspondences to generalized complex geometry. For example they share the same deformation theory. In this talk, I would like to introduce certain construction of compactified moduli spaces of NC del Pezzo surfaces. If time permits, I would also explain its relationship to the known constructions from the traditional noncommutative algebraic geometry and how (moduli of) elliptic curves (with additional structures) play role in our story. This talk is based on joint works with Kazushi Ueda (Tokyo) and Tarig Abdelgadir (ICTP).

**Alessandro Ottazzi** – *Sard Theorem for the endpoint map in sub-Riemannian manifolds.*

Abstract: Sub-Riemannian geometries occur in many areas of pure and applied mathematics, including harmonic analysis, PDEs, control theory, metric geometry, geometric group theory, and neurobiology. We introduce sub-Riemannian manifolds and give some examples. Therefore we discuss some

of the problems, and in particular we focus on the Sard Theorem for the endpoint map. This is related to the study of length minimizers, and we consider some new results obtained on this problem in collaboration with E. Le Donne, R. Montgomery, P. Pansu and D. Vittone.

**Daisuke Tarama** – *Some double fibrations arising from quadric line complexes.*

Abstract: A quadric line complex, which stands for the intersection of the projective image of the Grassmannian manifold  $\mathbf{Gr}(2,4)$ , consisting of 2-planes in  $\mathbb{C}^4$ , through Plücker embedding in  $\mathbb{C}\mathbb{P}^5$  and a generic quadric hypersurface, is one of the successful subjects of classical algebraic geometry. It has an essential relation to Kummer surfaces, typical examples of K3 surfaces. In this talk, the geometric structure of some double fibrations naturally arising from quadric line complexes is considered and the relation to classical physics is also mentioned.